5.3 Series Solutions near an ordinary Point II

Thm 5.3.1

Given $P(x)\,y'' + Q(x)\,y' + R(x)\,y = 0$, if $p = Q/P$ and $q = R/P$ are "analytic" at x_0 then x_0 is an ordinary point of the d.e. and

$$y = \sum_{n=0}^{\infty} a_n (x-x_0)^n = a_0 y_1 + a_1 y_2$$

where a_0 & a_1 are arbitrary, and y_1 and y_2 are linearly independent series solutions that are analytic at x_0.

[The radius of convergence of y_1 and y_2 is at least as large as the minimum of the radii of convergence of the series for p and q.]

Ex $\quad y'' + 4y' + 6xy = 0 \qquad x_0 = 0 \qquad$ Find lower bound of radius of convergence

$p(x) = 4 \qquad q(x) = 6x$

Both are analytic $\forall x$ so we can conclude that there is no lower bound.

Ex $\quad xy'' + y = 0 \qquad x_0 = 1 \qquad$ need to express q in terms of powers of $(x-1)$.

$p = 0\,, \quad q = 1/x = 1 + (1-x) + (1-x)^2 + (1-x)^3 + \cdots$

The interval of convergence for q is $|1-x| < 1$

so $\quad 0 < x < 2$

Since x_0 is right in the center of this we conclude $\underline{\rho = 1}$