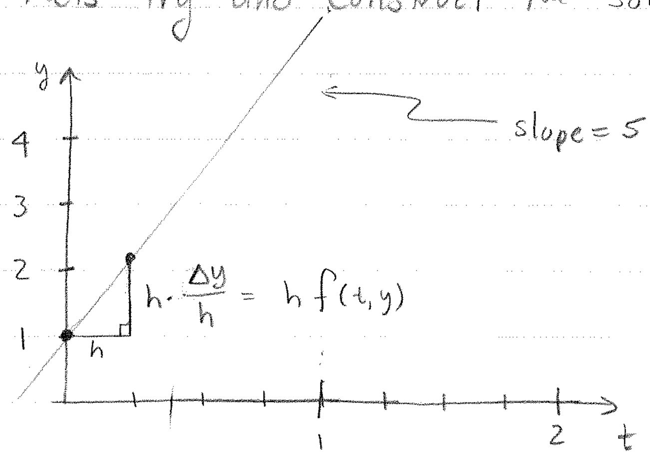
8.1 Eulers Method (Tangent Line Method)

Consider the IVP $\begin{cases} \frac{dy}{dt} = f(t,y) \\ y(t_0) = y_0 \end{cases}$

We will consider this solved if, on the interval in which the solution exists, we can obtain the value of y for any particular value of t.

Ex (Textbook) $y' = \underbrace{1 - t + 4y}_{f(t,y)}$ $\quad y(0) = 1$

Lets try and construct the solution graphically



t	y	y'
0	1	5
0.25	?	?

$$y(0.25) = y(0) + h f(0, y(0))$$

Basic idea is that we step from our initial point a distance h in the horizontal direction and $h f(t_0, y_0)$ in the vertical direction.

<u>Eulers Method</u>

Given t_0 and y_0 & stepsize h

$$y_{k+1} = y_k + h f(t_k, y_k)$$

$$t_{k+1} = t_k + h = t_0 + (k+1)h$$

Notice that we can obtain the formula for Euler's Method using a Taylor Series:

$$y(t_n+h) = y(t_n) + y'(t_n)\cdot h + \underbrace{y''(\bar{t}_n)\frac{h^2}{2!}}_{\text{Remainder term}}$$

for some $\bar{t}_n$ between t_n and t_n+h

If we ignore the last term $y''(\bar{t}_n)\frac{h^2}{2!}$ then we see that we have the Euler iteration formula

$$y(t_n+h) = y(t_n) + h f(t_n, y_n(t_n))$$

or $$y_{n+1} = y_n + h f(t_n, y_n)$$

There are other ways to generate this formula as well.

Consider $y' = 3 - 2y$ autonomous, 1st order, linear

Solution $y = -\frac{3}{2}(e^{-2t} - 1) + y_0 e^{-2t}$

Applying Euler method with $h = 0.1$, $y_0 = 1$

t	Exact	Euler	Error
0	1.0000	1.0000	0
1	1.4323	1.4463	-0.0140
2	1.4908	1.4942	-0.0034
3	1.4988	1.4994	-0.0006
4	1.4998	1.4999	-0.0001
5	1.5000	1.5000	$\lvert e \rvert < 1 \times 10^{-4}$