Order – the order of the highest derivative in the equation.

$$\frac{dx}{dt} = v(t) \qquad \underline{1^{st} \text{ order}}$$

$$\frac{d^2u}{dt^2} + \frac{du}{dt} = f(u,t) \qquad \underline{2^{nd} \text{ order}}$$

We can write a general n^{th} order equation as

$$F(x, u(x), u'(x), u''(x), \cdots, u^{(n)}(x)) = 0$$

or

$$F(x, y, y', y'', \cdots, y^{(n)}) = 0$$

where $y = u(x)$.

Solution – A solution of the ordinary differential equation $y^{(n)} = f(x, y, y', y'', \cdots, y^{(n-1)})$ on the interval $\alpha < x < \beta$ is a function ϕ such that $\phi', \phi'', \ldots, \phi^{(n)}$ exist and satisfy

$$\phi^{(n)} = f(x, \phi, \phi', \phi'', \cdots, \phi^{(n-1)})$$

for every x in $\alpha < x < \beta$.

NOTE: ϕ and all necessary derivatives must exist on the interval $\alpha < x < \beta$ and ϕ must satisfy the differential eq.

For example, consider $y''+y=0$.
It is easy to show that $y=\sin x+\cos x$ is a solution

$$\begin{aligned} y &= \sin x + \cos x & &\text{exists } \forall x \\ y' &= \cos x - \sin x & &\text{" "} \\ y'' &= -\sin x - \cos x & &\text{" "} \end{aligned}$$

So

$$y''+y = -\sin x - \cos x + \sin x + \cos x = 0 \quad \checkmark$$

Two questions arise

1. given a DE, does a solution <u>exist</u>?
2. If a solution exists, is it <u>unique</u>?

$y=\sin x+\cos x$ is not the only solution to $y''+y=0$.

And then the practical question,
assuming a solution exists, how do we find it?
(sometimes finding a solution is the easiest way to prove the existance of a solution).

<u>Linear Equation</u> – The equation $F(x,y,y',y'',\dots,y^{(n)})=0$ is <u>linear</u> if F is a linear function of $y, y', \dots, y^{(n)}$. This means that the equation can be written

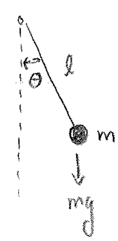
$$a_n(x)\,y^{(n)} + a_{n-1}(x)\,y^{(n-1)} + \dots + a_1(x)\,y' + a_0(x)\,y = g(x)$$

An eq. is <u>nonlinear</u> if it is not linear.

<u>Linear</u>	<u>nonlinear</u>
$y''+y=0$	$y'+y^2=0$
$x^2y'+\sin(x)\,y=\sqrt{x}$	$y'y=x^2$
$y'\sin x = x$	$x\sin y' = x$
Theory is well developed	Theory not so well developed

We live in a nonlinear universe but have made great strides using linear approximations.

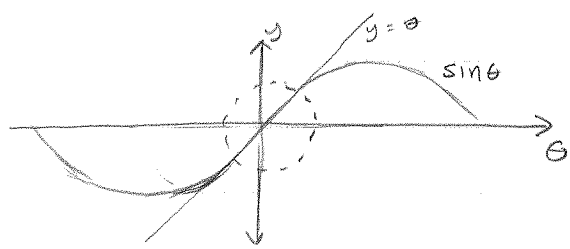
Classic example :



$$\frac{d^2\theta}{dt^2}+\frac{g}{\ell}\sin\theta=0$$

nonlinear because of $\sin\theta$

however, if θ is small (close to zero) then $\sin\theta \approx \theta$ (θ measured in radians)



So we obtain

$$\frac{d^2\theta}{dt^2}+\frac{g}{\ell}\theta=0$$

<u>linear</u>.