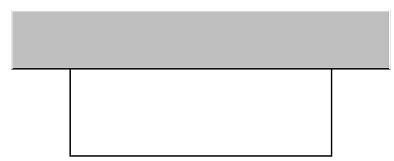
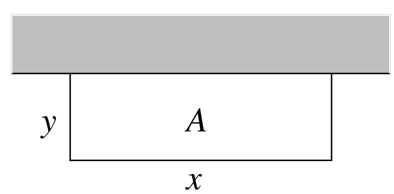
Example: Suppose you have 30 ft of fencing and want to fence in a rectangular garden next to a house. What should the dimensions of the rectangle be so that the garden is as large as possible?

Procedure	Solution
Read the problem, then read it again. Begin by reading the problem. It's okay if not everything makes sense at first, but try and get an idea of what the problem about and what it's asking you to do. What types of quantities are involved? What are their units?	Problem will involve distance (in feet) and area (units are sq. ft)
Draw a picture. In most cases you should start with a sketch. Don't worry about artistic quality; the point of the diagram is to help you visualize the problem, provide a place for you to label unknown quantities, and ultimately help you translate the problem into mathematics.	Here is a top-down view of the fenced garden. The shaded region represents the solid wall of the house. The rectangle next to it is the garden with the fence along the three sides that are not adjacent to the house. 
Name and label unknown quantities. Make up appropriate variable names for quantities you don't know and think you might need. Put these on your diagram.	Let x be the width and y be the height of the rectangle and use A for the area of the garden. The names used here are somewhat arbitrary. We could have just as easily used l and w for length and width. 
Determine what you're trying to optimize. Clearly identify the *objective*, which is the quantity you want to maximize or minimize.	In this problem we want to maximize the area, so our objective is A.
Develop an equation with the objective. Develop an equation that relates the objective to other known and unknown (but named) quantities. Use your diagram and pay close attention to units.	Area is length times width: $A = xy$.

Procedure	Solution
Write the objective in terms of a single variable. Use algebra and additional relationships to rewrite the objective equation so it expresses the objective as a function of a single variable.	Our objective has two variables, x and y and we want to eliminate one of them by expressing it in terms of the other. Recall that we were told the length of the fence was 30 ft, so $30 = x + 2y$. Solving for x we have $x = 30 - 2y$, so we can replace x in our objective with $30 - 2y$. $$A = (30 - 2y)y = 30y - 2y^2$$
Determine the domain. Clearly identify the set of values the independent variable can take on. The endpoints of this domain (if they exist) may correspond to extrema.	We note that area cannot be negative, so $A \geq 0$. Thus $(30 - 2y)y \geq 0$. This means $0 \leq y \leq 15$. The domain of our objective is the closed interval $[0, 15]$.
Find all extrema. Finally we get to use calculus! Usually this means to find the absolute extreme value of the objective. Remember, this means we need to identify and check all critical numbers as well as the endpoints of our domain.	Computing the derivative of A with respect to y we have $A' = 30 - 4y$. Since this is defined for all y we know that the only critical value occurs where $30 - 4y = 0$, which gives $y = 7.5$. We need to check three values for our absolute maximum: left endpoint $y = 0$, critical value $y = 7.5$, and right endpoint $y = 15$: $$A(0) = 0, \quad A(7.5) = 112.5, \quad A(15) = 0$$ So our absolute maximum is 112.5 and it occurs when $y = 7.5$.
Review the problem statement. Pay special attention to the form of the question or problem. Does your answer make sense? Are the units correct?	Our problem statement asked for the dimensions of the rectangle that provides the largest garden. We know one dimension, 7.5 ft, but still the other. Since $x = 30 - 2y$ we easily compute $x = 30 - 2 \cdot 7.5 = 15$ ft.
State your answer. Okay, you've done the math, now you need to communicate your answer clearly. Provide an answer for the problem using a **complete sentence**. *Note: The variable names you introduced when solving your problem should not appear in your answer!*	**Answer:** To maximize the garden area, the rectangle should be $7.5' \times 15'$, with the long side parallel to the house wall.

Note: Not all of the steps will be required to solve every optimization problem.