Basic Facts:

85% of taxis are Blue
15% are Green

Witness said the cab involved in the accident was Green

In a test of ability to distinguish colors of taxis at night, witness was right 80% of the time and wrong 20%

Question:

What is the probability that the taxi actually was Green?

We want P(taxi was Green | witness says it was)

or (to reduce writing)

P(Green | says Green)

where

"Green" is short for "The cab actually was Green"
"says Green" is short for "Witness says it was Green"

By Bayes' theorem:

$$P(\text{Green} \mid \text{says Green}) = \frac{P(\text{says Green} \mid \text{Green}) * P(\text{Green})}{P(\text{says Green})}$$

We can take P(says Green | Green) to be 0.8, since the witness correctly identifies the color of a cab at night 80% of the time

We take P(Green) to be 0.15, since 15% of the cabs in the city are Green

What is P(sys Green) - i.e. the probability that the witness would say it is Green - independent of what color it actually is?

P(says Green) =

P(was Green) * P(witness correct) +
P(was Blue) * P(witness wrong)

If an accident occurs involving a cab, absent any difference in driving skill between the drivers, we can take it that 85% of the time the cab is Blue and 15% Green.

We know that the witness identifies the color correctly 80% of the time and is wrong 20% of the time

So P(says Green) =

P(was Green) * P(witness correct) +
P(was Blue) * P(witness wrong) =

0.15 * 0.8 + 0.85 * 0.2 = 0.29

Therefore

$$\frac{P(\text{says Green} \mid \text{Green}) * P(\text{Green})}{P(\text{says Green})} =$$

$$\frac{0.8 * 0.15}{0.29} =$$

$$\frac{0.12}{0.29} =$$

0.41 as Dreyfus claimed